The Dynamics of the Age Structure, Dependency, and Consumption*

Heinrich Hock†
Florida State University

David N. Weil††
Brown University and NBER

March 24, 2006

Abstract

We examine the dynamic interaction of the population age structure, economic dependency, and fertility, paying particular attention to the role of intergenerational transfers. In the short run, a reduction in fertility produces a "demographic dividend" that allows for higher consumption. In the long run, however, higher old-age dependency can more than offset this effect. To analyze these dynamics we develop a highly tractable continuous-time overlapping generations model in which population is divided into three groups (young, working age, and old) and transitions between groups take place in a probabilistic fashion. We show that most highly developed countries have fertility below the rate that maximizes steady state consumption. Further, the dependency-minimizing response to increased longevity is to raise fertility. In the face of the high taxes required to support transfers to a growing aged population, we demonstrate that the actual response of fertility will likely be exactly the opposite, leading to increased population aging.

JEL Classification Numbers: E10, E21, H55, J11, J13
Keywords: Aging, fertility, intergenerational transfers, consumption

* This paper has benefited from the feedback provided by participants at the Southern Economic Association, the Brown University Department of Economics Macro Lunch and the Brown University Population Studies and Training Center Colloquium.

† Department of Economics, 288 Bellamy Bldg., Florida State University, Tallahassee, FL 32308-2180. Tel: 401-965-5699. E-mail: hhock@fsu.edu

†† Department of Economics, Box B, Brown University, Providence, RI 02912. Tel: 401-863-1754. E-mail: david_weil@brown.edu

1 Introduction

This paper examines the dynamic interaction of the population age structure, economic dependency and fertility, paying particular attention to the role of intergenerational financial transfers. Our goal is to anaylze how changes in fertility affect a country's demographic structure and how, via its effect on the dependency burden faced by working age adults, the age structure in turn affects fertility. We develop a highly tractable continuous-time overlapping generations model to analyze these dynamics. As will be seen below, none of the pieces that constitute our model is new. However, the feedback loop on which we focus is of great potential importance, and has so far not been studied by economists.

The effect of population age structure on economic outcomes is a widely studied topic. For example, Cutler et al. (1990) and Elmendorf and Sheiner (2000) discuss the effect of population aging in the United States on feasible and optimal paths of consumption. Because much of the transfer of resources to the elderly is channeled through governments, population aging will have a particularly dramatic effect on government finances (see Lee and Edwards, 2001). Bloom, Canning and Sevilla (2001) similarly examine how a "demographic dividend" resulting from reductions in fertility - that is, a period of several decades in which the ratio of working age adults to dependent children and elderly is unusually high - affects overall economic growth in developing countries.

In the above literature, the important chain of causality is from the demographic to the economic. That is, the underlying demographic inputs - most notably changes over time in fertility - are either taken as exogenous or related to phenomena (such as declining child mortality) that are outside the economic model being examined. In this paper we close the circle, and concentrate our attention on the interdependence of fertility and population age structure, through the channel of economic dependency. Specifically, we look at how changes in fertility affect population age structure and economic outcomes, and how these feed back to affect fertility.

A moment's consideration suggests that the problem that we are interested in is inherently dynamic. Changes in fertility have effects on the population age structure that take generations to play out. Most significantly, while the immediate effect of a decline in fertility is a reduction in a society's dependency burden, and so an expansion in the feasible level of consumption, the long-run effect of such a decline may be to actually raise dependency and lower consumption. Fertility and age structure are thus linked in a dynamical system. Such a system will have both steady states (in which fertility is constant and the population age structure is stable) and a long lasting dynamic response to external shocks.

Our interest in the mutual dependency of fertility and a society's age structure is primarily motivated by thinking about the future prospects of some of the most developed countries in the world. Countries such as Italy and Japan are now coming to the end of a decades-long period during which low fertility produced a transitorily low level of population dependency. During the period 2010-2030, rapid population aging will drastically impact consumption possibilities (and even more drastically impact government budgets). The effect of this consumption crunch on fertility - and thus on the age structure of the population even further down the road - is an issue that has not yet been addressed by economists and demographers.[1]

The link between population age structure and fertility that we examine is not the only such channel of causation. Most famously, Easterlin (1987) hypothesized that a

[1] Micevska and Zak (2002) examine fertility during income contractions that were even larger than those projected to result from population aging: The Great Depression (in the United States and Germany) and the transition from communism during the 1990s. In both cases, income declines produced significant reductions in total fertility. Micevska and Zak argue that such a phenomenon can be explained by a Malthusian model in which the perceived level of "subsistence" consumption is a function of an individual's own past consumption.

cohort's size was linked to its fertility through its effect on the earnings of young adults relative to those of their parents: members of a large cohort would find themselves with income that was low relative to the standard of living they had grown up with, and would adjust fertility downward to partially restore their standard of living. The mechanism that we examine here shares the feature of Easterlin's model that reductions in living standards trigger compensating reductions in fertility. However, our focus is on the fiscal effect of transfers to the elderly on after-tax earnings, rather than on the effect of cohort size on the pre-tax wage of young workers. For this reason, the key aspect of population age structure on which we focus is the ratio of elderly dependents to working age adults, rather than the ratio of older to younger workers on which Easterlin focuses. In principle the two mechanisms could easily coexist.

In addition to identifying the feedback from population age structure to fertility via the channel of dependency, a second contribution of this paper is to construct an analytically tractable dynamic model of population age structure. Specifically, we build a continuous time overlapping generations model in which population is divided into three groups (young, working age, and old), and transitions between groups take place in a probabilistic fashion *à la* Blanchard (1985).[2] Within this model, fertility can be taken as exogenous or made an endogenous function of the population age structure. The model is simple enough to be analyzed graphically, and yet captures the dynamic adjustment of age structure, fertility, and consumption to external shocks such as changes in old-age mortality, retirement age, or preferences regarding children. The model has applications well beyond those pursued here, and also represents a new and convenient way of conceptualizing demographic-economic interactions.

[2] Gertler (1999) extends Blanchard's model in a fashion similar to ours, but only allows for two age groups (working age and retired). Grafenhofer et al. (2005) present a model of probabilistic aging in which there are eight age groups, which they use to quantitatively assess the effects of population aging on savings, consumption, and the tax rate. Both models take fertility as exogenous.

The rest of this paper is structured as follows. In Section 2 we lay out our model of the population age structure and develop the basic dynamic equations that will be used in the subsequent analysis. Section 3 analyzes our model under the assumption that the rate of fertility is exogenous. We also show how the model can be used to calculate the rate of fertility that maximizes consumption in steady state and discuss the consumption-maximizing response of fertility to increases in elderly dependency. In section 4 we allow fertility to be an endogenous function of income, analyze the dynamics of the complete system, and compare the actual and consumption-maximizing responses of the economy to an exogenous shock to old-age mortality. Section 5 concludes.

2 A Dynamic Model of the Age Structure

Economists have long recognized the need to incorporate age heterogeneity into macroeconomic analysis. Samuelson (1958) and Diamond (1965) developed simple economic-demographic models in which agents with a finite life span progressed through a small and discrete progression of ages. Overlapping generations (OLG) models of this type have been used extensively in the economic growth literature. OLG growth models typically assume a two or three period lifecourse, which allows for a relatively clean analysis of fertility and old-age dependency. However, periods within such model represent long spans of time in the real world (e.g. 20-30 years). As a result, the dynamics generated are lumpy, with jumps in rates and stocks occurring only between model periods. This is satisfactory for an understanding of long-term transitions and equilibria. However, such models do not allow changes in fertility and the population structure to be analyzed over smaller time increments.

Adding more periods to a discrete time OLG model might allow for smoother dynamics, but adding more time periods implies adding more age groups. This increase in the state space implies difficulties in aggregation, making it more difficult to cleanly analyze macrostructural relationships. Under certain circumstance switching to continuous time may simplify matters. A classic example is Blanchard's (1985) "model of perpetual youth."[3] We develop here a somewhat stylized continuous-time model that borrows from both the traditional OLG framework and that of Blanchard. Given our interest in economic dependency, rather than focus on the lifecycle as a series of ages, we instead model it as a progression through a series of stages of economic life. Most individuals follow a pattern whereby they are first dependent on their parents, work for some amount of time, and then retire. Accordingly, we divide the population into three groups: A_Y is the stock of young people who have never worked; A_M is the stock of people in the economy who are in their working years; finally, A_O is the stock of people who once worked, but are now retired. The triple (A_Y, A_M, A_O) characterizes the *age structure* of the population.

Each individual i undergoes a monotone progression through the age structure. We apply the Blanchard idea to this demo-economic progression by assuming constant exit probabilities from each group A_j where $j \in \{Y, M, O\}$. For the young and working, λ_Y and λ_M give the hazard of transition to work and retirement, respectively. Among the elderly, λ_O is the probability of dying. All of the flows are determined by the structural parameters, $\{\lambda_j\}$, except the flow of births into A_Y, which is given by $N(t)$. The following system of equations summarizes this model of the evolution of the age structure:

$$\dot{A}_Y(t) = N(t) - \lambda_Y A_Y(t) \tag{1}$$

$$\dot{A}_M(t) = \lambda_Y A_Y(t) - \lambda_M A_M(t) \tag{2}$$

$$\dot{A}_O(t) = \lambda_M A_M(t) - \lambda_O A_O(t) \ . \tag{3}$$

[3] In a more recent paper, Bommier and Lee (2000) develop a model of an economy with a continuous age distribution, and are able to derive a number of interesting aggregate steady state results. However, they make only limited progress in analyzing aggregate dynamics, and even these are made under the assumption that the fertility rate is fixed at its steady-state value.

Table 1: Base Data for the OECD12

Country	R	T_M	T_O
Australia	62.7	42.7	20.9
Belgium	60.6	40.6	21.2
Canada	63.6	43.6	20.0
France	59.7	39.7	22.9
Germany	62.8	42.8	19.2
Italy	61.6	41.6	25.4
Japan	63.8	43.8	20.4
Netherlands	62.5	42.1	19.1
Spain	63.2	43.2	19.7
Sweden	63.9	43.9	19.4
United Kingdom	62.3	42.3	19.5
United States	64.4	43.9	18.3
Average	62.6	42.6	20.5

Data Sources: OECD (2004, Table SS8) and United Nations (2005, Tables 7 and 22).

We will use this system, under various assumptions regarding how fertility is determined, to characterize the evolution of the age structure.[4]

The parameters λ_j give the inverse of the average time spent in each age group, T_j. In referring to the number of members in A_j, we will use the term "size", and when discussing the average amount of time spent in group j,i.e. T_j we will call this the "width" of A_j. The widths of the age groups can be used to fit the model to demographic data.

[4] In our model we ignore childhood and early adult mortality, assuming that death only occurs among the retired. The model could easily be adapted to incorporate these other forms of mortality by changing the basic equations to:

$$\begin{aligned} \dot{A}_Y(t) &= n(t) A_M(t) - (\lambda_Y + \mu_Y) A_Y(t) && (\tilde{1}) \\ \dot{A}_M(t) &= \lambda_Y A_Y(t) - (\lambda_M + \mu_M) A_M(t) && (\tilde{2}) \\ \dot{A}_O(t) &= \lambda_M A_M(t) - \mu_O A_O(t). && (\tilde{3}) \end{aligned}$$

With this set-up, λ_j is the transition probability to the next group, and the μ_j is the death rate in the group. Analysis of this model would parallel that in the main text, with qualitatively similar results.

Since our paper is geared toward describing issues affecting the highly-developed world, we focus our attention to 12 members of the Organization for Economic Cooperation and Development (the "OECD12").[5] Since data on the age of entry into the labor force (T_Y) are difficult to come by, we assume that age 20 is a reasonable number. We set $T_M = R - 20$, where R is the average age of retirement and for OECD12 member nations is available from OECD (2004, Table SS8.1).[6] The United Nations (2005, Table 22) provides data on life expectancy in five-year increments. Since the decline in life expectancy is roughly linear between ages 55 and 75, we compute life expectancy at retirement (T_O) through linear interpolation of life expectancies at the surrounding five-year increments.[7] Table 1 presents the basic data (R, T_M, T_O) for the OECD12. As can be seen in the table, life expectancy after retirement is substantial. The proportion of life spent as an elderly dependent ranges from 0.22 in the United States to almost 0.3 in Italy.

2.1 Production and Dependency

In order to focus our attention on the dynamics of the age structure, we assume that output is produced solely by labor, which is supplied inelastically by people in their working years. The total pool of resources available for consumption is

$$\Omega(t) = W(t) A_M(t) \ ,$$

[5] These countries are selected so that they are among the top twenty in both GDP per capita and population among the OECD nations. The members of our OECD12 are: Australia, Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Spain, Sweden, the United Kingdom, and the United States.

[6] The measure of R is the average of the official and effective retirement ages, the latter of which is based on "observed changes in participation rates over a 5-year period for successive cohorts of workers (by 5-year age groups) aged 40 and over" (OECD 2004, notes accompanying Table SS8.1).

[7] Since our model does not account for sex differences, we compute R and T_O as the gender-weighted average of the retirement age and life expectancy values for each sex. Population weights are from United Nations (2005, Table 7).

where $W(t)$ is the prevailing wage at time t. The young and the elderly are supported through a system of transfers from the working. As a result, we focus on the youth and old-age dependency ratios:

$$y(t) = \frac{A_Y(t)}{A_M(t)} \text{ and } o(t) = \frac{A_O(t)}{A_M(t)}. \tag{4}$$

In the remainder of this section we establish various properties of the equations governing the evolution of the old-age and youth dependency ratios. These will serve as the basis for our subsequent analysis of dynamic equilibrium.

2.1.1 The old-age dependency ratio

The equation of motion for the old-age dependency ratio can be derived from (2) and (3) as

$$\dot{o}(t) = \lambda_M - (\lambda_O - \lambda_M)\, o(t) - \lambda_Y y(t)\, o(t). \tag{5}$$

The dynamics of $o(t)$ in (y, o)-space are relatively straightforward, and we present some basic results here that are applicable throughout the paper.

Based on (5), the zero motion locus for $o(t)$ is given by

$$o(t)|_{\dot{o}=0} = \frac{\lambda_M}{(\lambda_O - \lambda_M) + \lambda_Y y(t)} \equiv Z_o(y(t)). \tag{6}$$

Hence, for a given value of the youth dependency ratio, there is exactly one equilibrium value of o. Considering the graphical properties of the $Z_o(y)$ in (y, o)-space, the vertical intercept is given by $\lambda_M / (\lambda_O - \lambda_M) > 0$. That the intercept is positive follows from the fact that the expected time in retirement is less than the expected time working. Thus, since $T_O < T_M$, we have that $\lambda_O > \lambda_M$. Because $Z_o'(y) < 0$ and $Z_o''(y) > 0$, the zero motion locus for is downward sloping and convex with respect to the origin.

Finally, noting that $d\left[\dot{o}(t)\right]/d\left[o(t)\right] < 0$, all equilibrium values of o are stable. That is, for a given value of the youth dependency ratio, the old-age dependency ratio steadily converges to the corresponding point on the zero motion locus.

2.1.2 Fertility and the youth dependency ratio

The dynamics of the youth dependency ratio requires depend on the flow of births, which is a function of the stock of fertile persons and the rate of fertility among the fertile. We give here a fairly general dynamic equation for the youth dependency ratio, laying out a basic assumption regarding the stock of fertile of persons. In later sections we specify the rate of fertility among the fertile in greater detail.

From (1) and (2) the equation of motion for the youth dependency ratio can be written

$$\dot{y}(t) = \frac{N(t)}{A_M(t)} - (\lambda_Y - \lambda_M)\, y(t) - \lambda_Y \left[y(t)\right]^2. \tag{7}$$

For simplicity, we assume that the fertile population is a fixed proportion, ϕ, of the workforce. Since all members of the workforce are homogenous in our model, an alternative interpretation of ϕ is to decompose it as $\phi = T_F/T_M$, where $T_F < T_M$ is the expected length of time spent fertile among each member of M.[8] Letting $n(t)$ represent the average fertility rate at time t among the fertile, under this assumption (7) reduces to

$$\dot{y}(t) = \phi n(t) - (\lambda_Y - \lambda_M)\, y(t) - \lambda_Y \left[y(t)\right]^2. \tag{8}$$

We will refer to $\phi n(t)$ as the flow of births per worker.

The equation of motion for the youth dependency ratio (8) does not depend explicitly

[8] An additional set of equations, consistent with those governing the economic age structure, could be specified to allow ϕ to vary over time. However, doing so makes analyzing the model substantially more complicated, while adding very little qualitatively to our analysis.

on $o(t)$. Thus, if fertility is determined in a manner unrelated to the old-age dependency ratio, equilibrium analysis of the relative economic age structure is straightforward. This is the case in Sections 3 where we take the fertility rate as given. In Section 4 we allow fertility to be endogenously determined. Since fertility will be a function of after-tax income, transfers to the elderly through the pension system cause the old-age dependency ratio to play a role in the dynamics of the youth dependency ratio.

3 Fertility and Economic Dependency

In this section we analyze our model assuming that the fertility rate is fixed and exogenous. (That is, $n(t) = n$ for all t.) We focus on the relationship between fertility, old-age mortality and a measure of total dependency in the economy. The traditional demographic measure of total dependency is simply the sum of youth dependency, y, and old-age dependency, o. However, as pointed out by Cutler et al. (1990), the consumption requirements of the young and the elderly are not necessarily the same. This implies that an increase in youth dependency will not generally have the same implications for resource availability as an increase in old-age dependency. Consequently, we focus on a measure of needs-weighted economic dependency, $e(t)$. In particular we assume that the children and elderly have consumption needs equivalent to ρ_Y and ρ_O times that of the working, which implies that economic dependency at a given moment is

$$e(t) = \rho_Y y(t) + \rho_O o(t) \ . \tag{9}$$

Given our labor-based production structure, we assume further, as in Weil (1999), that the consumption of the all three groups is indexed to the wage. That is $c_Y(t) = \rho_Y c_M(t)$,

$c_O = \rho_O c_M(t)$, and $c_M(t) = \eta(t) W(t)$. Given the aggregate resource constraint,

$$c_Y(t) A_Y(t) + c_M(t) A_M(t) + c_O(t) A_O(t) = \Omega(t) \ ,$$

the consumption index must satisfy

$$\eta(t) = \frac{1}{1 + e(t)} \, . \tag{10}$$

The index η affects the consumption of all groups proportionately. In the parlance of Cutler et al., η is the "support ratio," that is, the ratio of the production capacity of the economy to the consumption needs of the population. Increases in youth and elderly dependency reduce the per-capita resources in the economy, but their effects on η are proportional to the relative consumption needs of the young and old. Below we consider the impact of changes in the fertility rate on economic dependency and consumption. We then consider the consequences population aging due to increased life-expectancy among the elderly. Before proceeding we characterize the basic equilibrium of in our model, given constant rate of fertility.

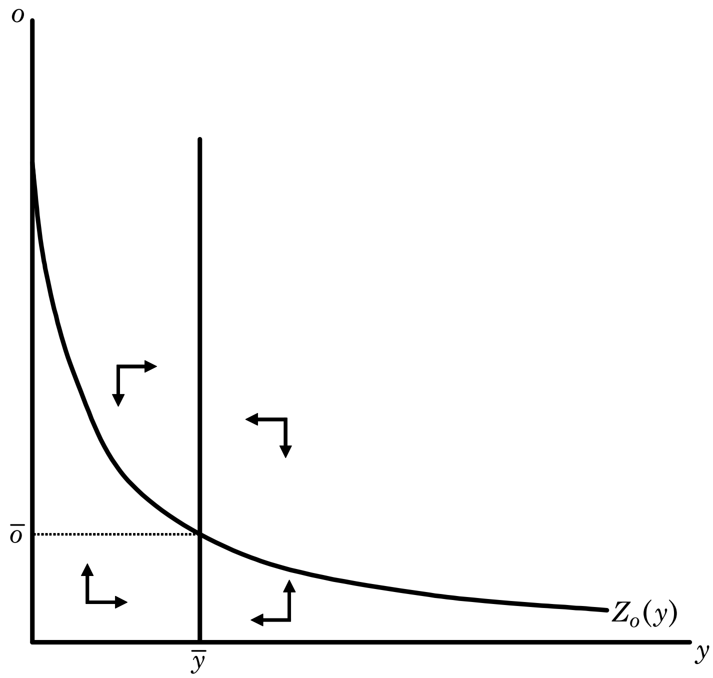
3.1 Equilibrium with Exogenous Fertility

With a fixed fertility rate, the equation of motion for the youth dependency ratio is

$$\dot{y}(t) = \phi n - (\lambda_Y - \lambda_M) y(t) - \lambda_Y [y(t)]^2 \, . \tag{11}$$

Since the dynamic equation does not depend on $o(t)$, (11) can simply be examined for equilibrium values of the youth dependency ratio. Setting $\dot{y}(t)$ equal to zero yields one positive root, denoted by $\bar{y}$. Writing this in terms of the widths of the age groups (T_j),

Figure 1: Global Dynamics with Exogenous Fertility



rather than the exit parameters (λ_j),

$$\bar{y} = -\frac{1}{2}\left(1 - \frac{T_Y}{T_M}\right) + \sqrt{\left(\left[\frac{1}{2}\left(1 - \frac{T_Y}{T_M}\right)\right]^2 + T_Y \phi n\right)}. \tag{12}$$

The equation of motion for y implies that this root is stable. The corresponding equilibrium old-age dependency ratio is given by

$$\bar{o} = Z_o(\bar{y}) = \frac{1}{(T_M/T_O - 1) + (T_M/T_Y)\,\bar{y}}. \tag{13}$$

Graphically, Figure 1 portrays the global dynamics of the population dependency ratios. The equilibrium point is globally stable and the population dependency ratios converge monotonically to the point $(\bar{y}, \bar{o})$.

It is convenient to consider the flow of births per worker in terms of the gross reproductive rate (GRR), a fertility concept commonly used by demographers. Since individuals in our model are the reproductive unit of analysis, the GRR is the number of children an individual can expect to have, assuming that they are subject to the currently-prevailing rate of fertility for the entirety of their childbearing years. Given our assumptions regarding the stock of fertile persons, this implies that the gross reproductive rate is simply $G = T_F n$, and that the flow of births per worker is

$$\phi n = \frac{G}{T_M}\,, \tag{14}$$

Note that when G equals one, the population of workers is just replacing itself. Substituting in (14) for the flow of births, the equilibrium youth dependency ratio can be rewritten in terms of the GRR:

$$\bar{y} = -\frac{1}{2}\left(1-\frac{T_Y}{T_M}\right)+\sqrt{\left[\frac{1}{2}\left(1-\frac{T_Y}{T_M}\right)\right]^2+\frac{T_Y}{T_M}G}. \tag{15}$$

When the GRR is at the replacement rate the equilibrium dependency ratios simplify to $\bar{y}^r = T_Y/T_M$, and $\bar{o}^r = T_O/T_M$. These results are fairly intuitive, and parallel those in basic demographic theory. In a stable population with replacement fertility, the relative sizes of the age groups in equilibrium should simply be the ratios of their widths.

3.2 Declining Fertility and the Consequences for Consumption

Much of the twentieth century was characterized by declines in period fertility rates, with the exception of the post-World War II baby boom. Considering the implications for the population dependency ratios, given a fall in G, equation (15) implies that the equilibrium youth dependency ratio falls. As shown in Figure 2, the decline in $\bar{y}$ maps to a higher equilibrium value of the old-age dependency ratio along the $Z_o(y)$ locus.

Considering the transition to an equilibrium with lower fertility, initially the declines in youth dependency are larger than the increases in old-age dependency, so that economic dependency (e) declines. This leads to an increase in the consumption index η, reflecting the "demographic dividend" discussed by Bloom, Canning and Sevilla (2001). Later in the transition, there comes a turning point where the declines in e caused by y are more than offset by the increases in e, in which case η begins to increase as the economy approaches the new equilibrium.[9] Hence, the consumption benefits from the demographic dividend are time-limited. The net effect of a fertility decline on equilibrium economic dependency is *a priori* ambiguous. The inset of Figure 2 shows a possible set of paths for y, o, and η after a decline in the fertility rate in the case where the new equilibrium has a higher level of economic dependency (and a lower level of consumption) than the original equilibrium. As we will see, this likely represents the effects of further fertility declines in most of the highly developed world.

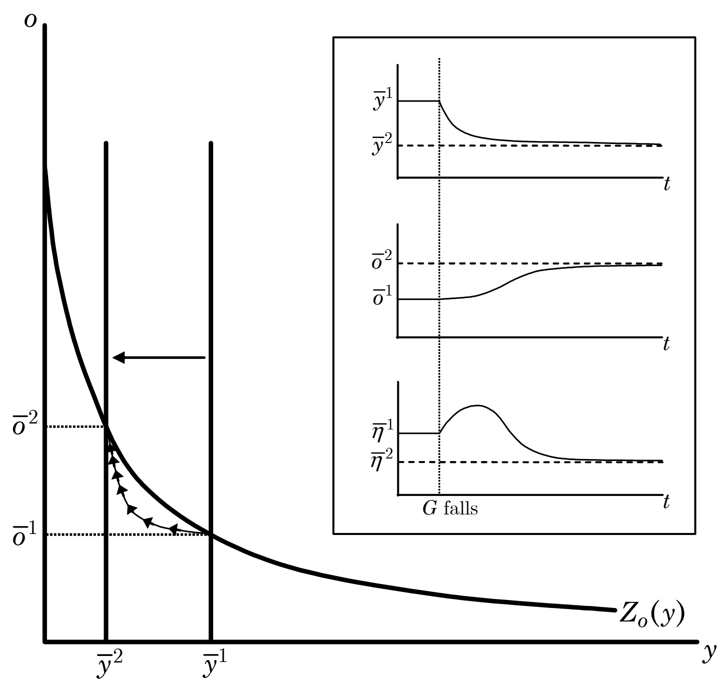
Following Weil (1999), given a certain value of economic dependency, $\tilde{e}$, we can use (9) to graph iso-dependency lines in (y, o) space as

$$o = \frac{\tilde{e}}{\rho_O} - \frac{\rho_Y}{\rho_O} y \equiv I_{\tilde{e}}(y) \tag{16}$$

Since changes in fertility trace out equilibrium population dependency ratios along $Z_o(y)$, we may consider the relationship between the iso-dependency lines and this locus. Iso-dependency lines that are closer to the origin are associated with a lower rate of economic dependency and a higher consumption index, η. Because $Z_o(y)$ is convex with respect to the origin, there is a unique equilibrium pair $(\bar{y}^m, \bar{o}^m)$ that is tangent to the $Z_o(y)$ locus. (See Figure 3.) The pair $(\bar{y}^m, \bar{o}^m)$ minimizes economic dependency, which implies

[9] The initial decline in e should be evident from an analysis of the phase diagram. While o is still close to the $Z_o(y)$ locus, y is very far from its steady state. Our assertion of a turning point makes intuitive sense, and can be established based on numerical simulations of the model using parameter values describing the OECD12 nations.

Figure 2: The Effects of a Decline in the Fertility Rate

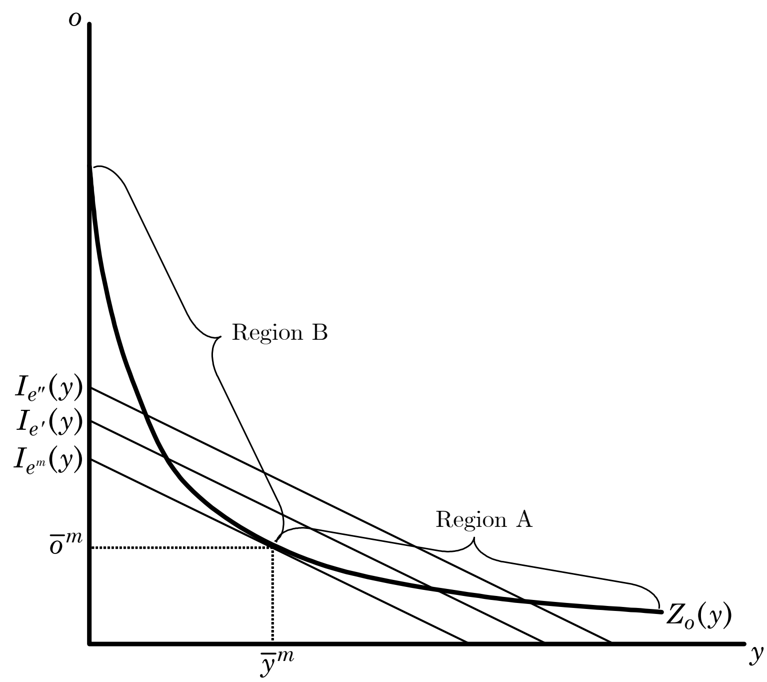


that consumption of all members of the economy is maximized. At this point, the slope of the $Z_o(y)$ locus must equal the slope of the iso-dependency line, which based on (13), (15), and (16) implies that

$$\bar{o}\left(G, \frac{T_Y}{T_M}, \frac{T_O}{T_M}\right) = \sqrt{\left(\frac{\rho_Y}{\rho_O}\right)\frac{T_Y}{T_M}}. \tag{17}$$

Given that $\bar{o}$ is a monotonically negative function of G, there is a unique fertility rate, G^m, that generates this equilibrium, given values of the other parameters. At equilibria corresponding to higher fertility rates, i.e. those in Region A of the $Z_o(y)$ locus in Figure 3, small declines in G are associated with lower equilibrium economic dependency and higher consumption. The opposite occurs in Region B, which is associated with lower fertility rates.

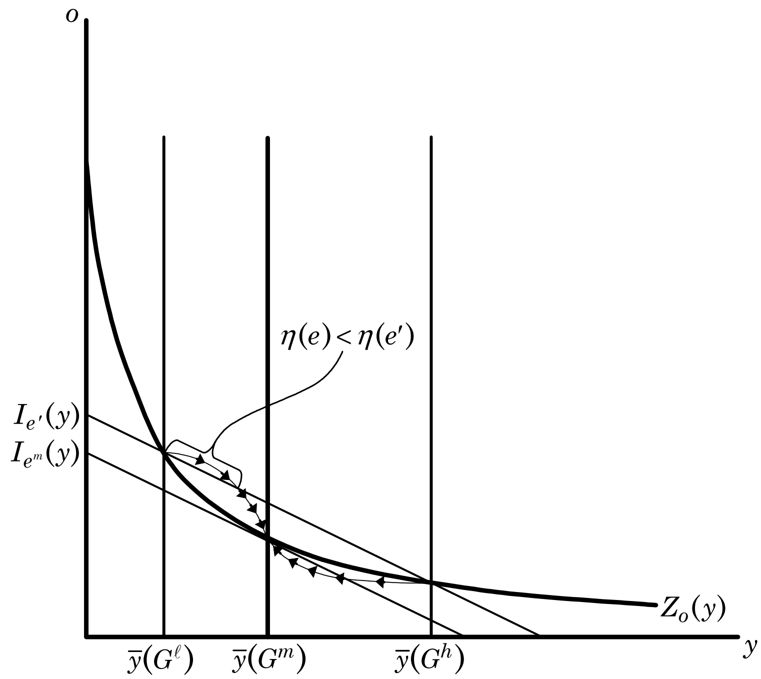
Figure 3: Economic Dependency and the $Z_o(y)$ Locus



For a country with a fertility rate $G^h > G^m$ (i.e. in Region A), as the fertility rate declines toward the consumption-maximizing rate, consumption increases at all future times, assuming that the other parameters remain stable. On the other hand, for a country with a fertility rate below the consumption-maximizing rate the transition to G^m is costly. As shown in Figure 4, in a country with the same initial economic dependency as implied by G^h and a fertility rate of $G^\ell < G^m$, economic dependency must initially increase, implying a reduction in η. It is only later in the transition, as the elderly dependency ratio declines, that the consumption benefits from the increase in fertility are realized.

Given a set of widths of the age groups and relative consumption needs, ascertaining whether an economy is in Region A amounts to checking whether the observed fertility rate (G^a) is at or above the consumption-maximizing fertility rate. We undertake this

Figure 4: Transition to Consumption-Maximizing Equilibrium from Above and Below



calculation for the OECD12 nations. Based on measures of education, health and private consumption expenditures, Elmendorf and Sheiner (2000) estimate that $\rho_Y = 0.62$ and $\rho_O = 1.37$ in the United States.[10] It seems reasonable that ρ_Y and ρ_O should follow a similar pattern in the other highly-developed nations. Taking the values of ρ_Y and ρ_O from Elmendorf and Sheiner, and using the base data on age widths from Table 1, we compute G^m for the OECD12 nations based on equation (17). In the first column of Table 2, we report the actual gross reproductive rate and in the second we give our calculation of G^m. In eleven out of the twelve nations, the current GRR is less than the consumption-maximizing fertility rate, and for most the difference is quite substantial. It is only in the United States that the currently-observed fertility rate exceeds G^m.[11] Hence,

[10] These update the estimates of the relative consumption needs given by Cutler et al. (1990).

[11] That the consumption-maximizing rate is so low in the U.S. is largely a function of the relatively short life expectancy after retirement. Similarly, G^m is the highest in Italy where life expectancy after retirement is the longest. We discuss the relationship between G^m and life expectancy more explicitly below.

Table 2: Actual and Consumption-Maximizing Gross Reproductive Rates and Potential Consumption Gains for the OECD12

Country	G^a	G^m	Potential gain in η (%)
Australia	0.88	1.19	0.41
Belgium	0.83	1.33	1.36
Canada	0.76	1.03	0.40
France	0.94	1.61	1.53
Germany	0.66	0.91	0.49
Italy	0.64	1.88	6.58
Japan	0.67	1.07	0.25
Netherlands	0.86	0.91	0.05
Spain	0.64	0.98	0.74
Sweden	0.82	0.91	0.06
United Kingdom	0.83	1.00	0.11
United States	1.02	0.72	0.26
Average	0.80	1.13	1.02

Notes: G^a is from United Nations (forthcoming); G^m and the percentage gain in the consumption index, η, are calculated as described in the text.

based on present demographic data, the equilibria for most highly-developed economies fall in Region B of the $Z_o(y)$ locus. This suggests that recent declines in fertility will ultimately be associated with higher equilibrium rates of economic dependency and lower levels of consumption. Conversely, equilibrium consumption could be increased in most countries through increases in the fertility rate.

In order to determine the equilibrium benefits from moving to the consumption-maximizing rate of fertility, we use equations (13) and (15) to calculate the steady-state population dependency ratios for the OECD12 consistent with the age widths in Table 1 and the values of G^a and G^m reported in Table 2. Weighting these by the relative consumption needs gives us the equilibrium level of economic dependency associated with the actual fertility rate and with the consumption-maximizing fertility rate. We denote these as $\bar{e}^a$ and $\bar{e}^m$, respectively. Finally, equation (10) allows us to compute the per-capita consumption index η associated with these two values. The third column of Table 2 reports the percentage increase in $\eta(\bar{e}^m)$ relative to $\eta(\bar{e}^a)$. The average potential steady-state consumption gain in the OECD12 is approximately one percent. Countries with fertility rates furthest away from their respective values of G^m obviously have the most to gain. For example, if the fertility rate in Italy were to rise to its consumption-maximizing rate, the steady-state per-capita consumption index would rise by almost 6.6 percent. However, since most of the countries under consideration have fertility rates below their consumption-maximizing rate, the transition to G^m would entail a transitory period of higher economic dependency and lower consumption.

A number of explanations have been proposed for the currently low levels of fertility in the most-developed nations, which we have so far taken as given.[12] Nonetheless, the future path of fertility in these "lowest-low fertility" countries is a matter of some debate among demographers, and there is limited theory to guide predictions. At the same time, it is almost a certainty that old-age mortality will decline in the future. In Section 3.3, we explore the consequences of falling mortality among the elderly in terms of economic dependency and consumption, and in Section 4 we consider a mechanism whereby increased life expectancy itself may affect the path of fertility.

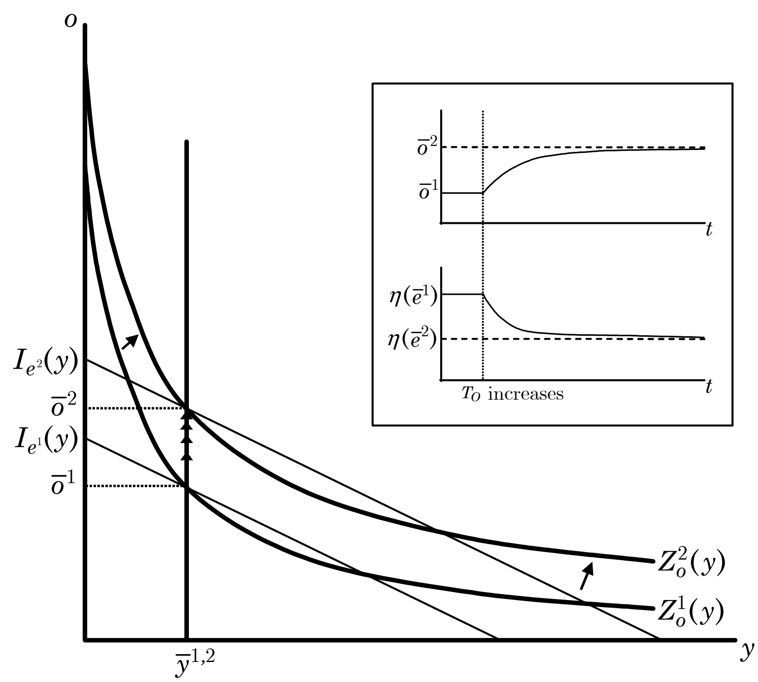
3.3 The Effects of Falling Old-Age Mortality

Old-age mortality has fallen substantially over the past fifty years, and is expected to decline further in the future. In the United States, for example, life expectancy at age 65 has risen by 3.25 years since 1955 and is expected to rise by approximately the same amount over the next 50 years.[13] Other highly-developed countries have seen comparable

[12] See, e.g. Kohler, Billari and Ortega (2002).

[13] Here we use the simple, rather than gender-weighted, average of sex-specific life-expectancies. The data on period life expectancies and come from the Board of Trustees, Federal Old-Age and Survivors

Figure 5: The Effects of a Decline in Old-age Mortality with Fertility Held Constant



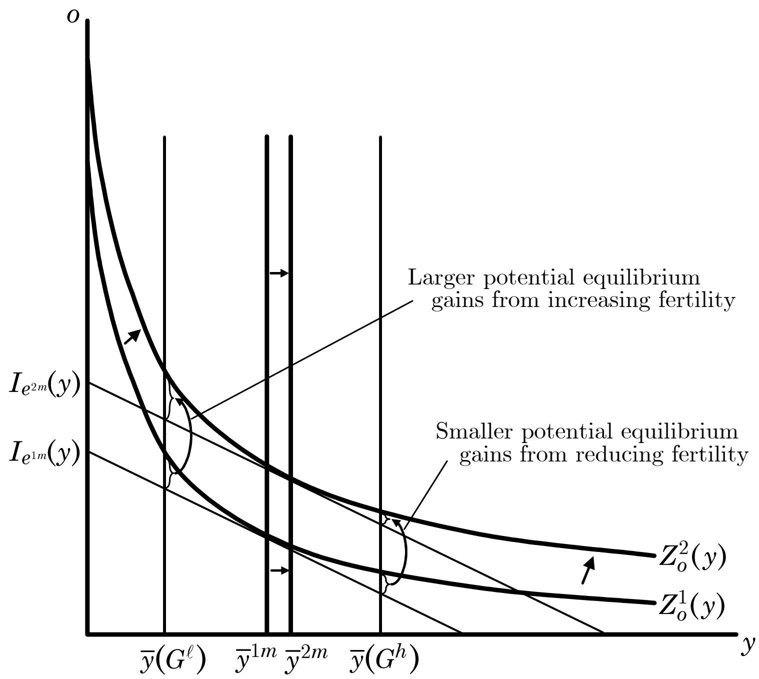
or greater reductions in old-age mortality over the past half-century and can be expected to follow a similar trend in the future.[14] Assuming a constant rate of fertility and constant values of the other parameters, Figure 5 demonstrates the effects of an increase in life expectancy among the elderly in the context of our model. As life expectancy rises from T_O^1 to T_O^2, the $Z_o(y)$ locus shifts upward, leaving youth dependency unchanged. The old-age dependency ratio increases monotonically toward the new equilibrium, while the consumption index η declines due to the higher burden of elderly dependency.

The consumption-maximizing fertility rate is a function of the life expectancy of the elderly, as described in equation (17). *Ceteris paribus*, as life expectancy rises so does G^m because the total cost of support per elderly dependent rises. While an increase in fertility

Insurance and Disability Insurance Trust Funds (2005, Table V.A3). For the projected gain in life expectancy, we use the "intermediate" variant. Other independent mortality projections (e.g. Lee and Carter 1992) predict even higher values of life expectancy.

[14] See, e.g., Munnell, Hatch and Lee (2004).

Figure 6: The Effects of a Decline in Old-age Mortality on the Consumption-Maximizing Equilibrium



would cause y to rise in equilibrium, the reduction in η due to higher youth dependency is more than offset by the eventual decrease in the more consumption-intensive elderly dependency. For countries that initially have a fertility rate higher than G^m (such as G^h in Figure 6), the consumption-maximizing rate draws closer to their existing fertility rate. As seen in the figure, not only does the best-attainable level of consumption fall, so does the potential equilibrium gain from reducing the fertility rate.[15] For countries with a fertility rate lower than G^m (e.g. G^ℓ in Figure 6), the consumption-maximizing fertility rate moves further away as old-age mortality falls. Although the best-attainable level of consumption falls, the potential equilibrium gains from a higher fertility rate are greater.

[15]This can be established analytically since the $Z_o(y)$ locus gets steeper for any value of y as T_O increases.

Repeating the numerical calculation from Section 3 for all OECD12 under the assumptions that T_O increases by 3.25 years and that actual fertility rates remain unchanged results in an average potential increase in η of just over 3 percent. Further, the United States joins the eleven other nations in having a fertility rate below the consumption-maximizing rate. As we will see below, the burden of old-age dependency itself may cause fertility rates to decline (rather than rise), leading to long-run outcomes that are even less efficient from a consumption standpoint.

4 Old-Age Dependency and Fertility

In our analysis of economic dependency and consumption thus far, we have taken the fertility rate as exogenous. This ignores the vast economics literature that regards fertility, for the most part, as the result of a decision making by potential parents who weigh the costs and benefits of having children. Hence, in addition to differences in consumption needs, there is a more profound asymmetry between youth dependency and elderly dependency. The economic burden of youth dependency is partially the result of the fertility choices of the working. On the other hand, adults have no choice over the number of elderly persons in the economy, but are typically required to support them through public pension systems. In a pay-as-you-go pension system, rising elderly dependency reduces the per-capita resources available to the working. Given standard economic models of child-bearing decisions, this potentially provides a partial explanation for declines in the fertility rate over the last half-century. Drucker (1990, p.7) takes a stronger stand on this issue, arguing that the primary cause of low and declining fertility in the developed world is that "its younger people are no longer able to bear the increasing burden of supporting a growing population of older, nonworking people."

Boldrin et al. (2005) provide some empirical evidence in support of this idea, employing a panel sample from 1960 to the present for eight Western European countries. Controlling for measures of the generosity of the pension system and infant morality, they find a strong and negative relationship between fertility and the share of the population 65 and over (p_{65}). In the eight countries in their sample the average increase in p_{65} between 1960 and 2000 was approximately 4.7 percentage points.[16] Applying their regression coefficient to this difference implies a predicted decline of 0.15 in the GRR, which was over a quarter of the average actual reduction in the GRR in the sample. Thus, increases in elderly dependency potentially explain a good deal of the recent declines in fertility.

We develop here an extension to our basic model that allows this sort of dynamic interaction between old-age dependency, fertility and the age structure. We retain the production structure described above, where workers provide labor inelastically in return for a wage of $W(t)$. Wages are subject to a proportional tax τ, which is used to fund public transfers to the old and to the young. We assume that the elderly are supported entirely through public transfers in the form of a pension system, while the young receive both public support and direct transfers from their parents. Although somewhat stylized, this distinction mirrors the substantial difference in the pattern of transfers to the old and young observed in reality. According to Mason et al. (2005), private transfers account for over 61% of the transfer-based consumption of persons under the age of 20 in the United States, but only 11% of the total transfers to persons over the age of 65.

The pension system replaces after-tax wages at a rate β, and we assume that per-capita public transfers to the young are equal to a fixed proportion, π, of the net transfers

[16] Our data on the share of the population age 65+ and fertility rates drawn from United Nations (2000 and forthcoming).

to the elderly. The tax rate at time t is determined by the aggregate resource equation

$$A_Y \alpha W(t)(1-\tau(t)) + A_M(t) W(t)(1-\tau(t)) + A_O \beta W(t)(1-\tau(t)) = \Omega(t) , \quad (18)$$

where $\alpha = \pi\beta$. Solving (18) for the balanced-budget tax rate gives

$$\tau(t) = \frac{\alpha y(t) + \beta o(t)}{1 + \alpha y(t) + \beta o(t)} .$$

Higher rates of youth and old-age dependency both increase the tax rate. As a result, increases in both forms of dependency decrease the take-home pay of the workers and the size of per-capita public transfers to the youth and elderly. Given values of α and β, we can solve for the tax-rate minimizing gross reproductive rate, G_τ^m. Similar to the analysis in Section 3, G_τ^m is the implicit solution to

$$\bar{o}\left(G_\tau^m, \frac{T_Y}{T_M}, \frac{T_O}{T_M}\right) = \sqrt{\frac{\alpha}{\beta}\frac{T_Y}{T_M}} . \quad (19)$$

Because of the large share of the consumption of children that is privately funded, (α/β) will be substantially less than (ρ_Y/ρ_O), which plays a corresponding role in equation (17). Since $\bar{o}$ is a negative function of G, this will imply that the tax-rate minimizing rate of fertility is substantially higher than that which maximizes consumption in the framework of Section 3. We do not attempt to quantify G_τ^m, focusing instead on the divergence between the privately chosen rate of fertility and the tax-minimizing rate.

At each moment in time the fertile population of workers allocate their after-tax income to their own consumption and to bearing children. The price of goods and services is numeraire. The nature of our model of the age structure makes it difficult to link children to their parents, so we assume that childrearing expenses are paid up front into a trust fund. Further, the consumption needs of children at any given moment are indexed to the prevailing wage, equalling χW. This implies that, conditional on a given level of fertility and a tax rate (i.e., age structure), the consumption of the parents will also be multiple of the pre-tax wage.[17] With a constant rate of wage growth, g, to ensure that children always receive the requisite consumption, an actuarially-neutral trust fund will set the price of children as

$$p_n = \xi W(t) \ ,$$

where $\xi = \chi T_Y / (1 - gT_Y)$.[18]

For simplicity we assume a log utility function, which implies fertile workers face the following optimization problem

$$\max_{c(t),n(t)} \ln [c(t)] + \theta \ln [n(t)] \tag{20}$$

subject to

$$c(t) + \xi W(t) n(t) = w(t), \tag{21}$$

where $c(t)$ is the consumption, $n(t)$ is the annual flow of births per fertile worker, and $w(t) = [1 - \tau(t)] W(t)$ is the take-home wage of the working. Solving for the flow of births

$$\tilde{n}(t) = \psi \frac{w(t)}{W(t)} = \frac{\psi}{1 + \alpha y(t) + \beta o(t)}, \tag{22}$$

where $\psi = \theta/(\xi(1+\theta))$. The flow of births per fertile worker satisfies some usual properties: fertility is a positive function of the relative preference for children (θ) and a

[17] The consumption requirement of children could be directly indexed to that of the parents, and we could also include a time cost of children. Incorporating these would yield qualitatively similar results to the model we present, so we ignore them for ease of exposition.

[18] For the path of trust fund payments to remain bounded, g must be less than $1/T_Y$. Taking 25 years as an upper limit on the average length of youth dependency, this implies that g must be less than 4% over the long run, which is quite reasonable.

negative function of childrearing costs (ξ).

The privately-optimal fertility rate is inefficient relative to the that which would be chosen by a social planner whose goal is to maximize the equilibrium utility of fertile workers. To see this, consider the optimization problem of the social planner who takes into account the both the utility that fertile workers get from fertility and consumption as well as the effects of fertility decisions on the dependency ratios. Substituting in the budget constraint (21) into the objective function (20), the social planner's problem can be written

$$\max_{\hat{n}} \ln \left[\frac{1}{1 + \alpha y(\hat{n}) + \beta(\hat{n})} - \xi \hat{n} \right] + \theta \ln [\hat{n}] \ . \tag{23}$$

The first order condition for (23) can be written

$$\hat{n} = \tilde{n}(\hat{n}) + \frac{1}{\xi(1+\theta)} \frac{[-\alpha y'(\hat{n}) - \beta o'(\hat{n})] \hat{n}}{(1 + \alpha y(\hat{n}) + \beta o(\hat{n}))^2} \, , \tag{24}$$

where $\tilde{n}(\hat{n})$ is the fertility rate that workers would chose, conditional on the equilibrium $(o(\hat{n}), y(\hat{n}))$. Based on the equilibrium equations (12) and (13), the term in brackets on the right-hand side of (24) is equal to

$$[-\alpha y'(\hat{n}) - \beta o'(\hat{n})] = \left[-\alpha + \beta [o(\hat{n})]^2 (T_M/T_Y)\right] y'(\hat{n}) \ .$$

Since $y'(\cdot)$ is negative, the social planner's chosen fertility rate, $\hat{n}$, is higher than the worker's private choice based on the age structure implied by $\hat{n}$ when

$$\beta [o(\hat{n})]^2 (T_M/T_Y) - \alpha < 0 \, ,$$

and is lower otherwise. Consequently, $\hat{n}$ equals $\tilde{n}(\hat{n})$ only when (19) holds, which implies that $\hat{n} = n^m_T$. That is, a social planner optimizing the fertile workers' equilibrium utility would set the fertility rate equal to that which minimizes the tax rate. As old-age mortality declines, there will be a greater divergence between privately-chosen fertility rates and the tax-minimizing rate.

The dynamic analysis of Section 3 can easily be augmented to analyze the effects of increased life expectancy among the elderly. Due to a standard income effect, changes in o affect fertility through the tax rate. As a result, the dynamics of the youth dependency ratio will be affected by the old-age dependency ratio. Putting the flow of births per fertile worker, $\tilde{n}$, into the equation of motion of the youth dependency ratio (equation (8)), the graph of the zero-motion locus of y in (y, o)-space is

$$o|_{\dot{y}=0} = \frac{1}{\beta}\left[\frac{\phi\psi T_Y}{(1 - T_Y/T_M)\, y + y^2} - (1 + \alpha y)\right] \equiv Z_y\left(y\right) \, .$$

This locus represents equilibrium values of y that are stable (because $d\dot{y}/dy < 0$). Further, it is downward sloping and convex with respect to the origin, asymptotes to infinity as y approaches zero, and crosses the horizontal axis for large enough values of y. Finally, there is a single crossing in the positive orthant between the $Z_y\left(y\right)$ and $Z_o\left(y\right)$ loci, which implies that a globally stable equilibrium in the population dependency ratios, depicted in Figure 7.[19]

We depict the transition associated with a fall in mortality in this model of endogenous fertility in Figure 8. As discussed above, lower rates of old-age mortality translate directly into increases in the rate of elderly dependency due to longer life expectancy after retirement. If fertility rate were to remain unchanged, old-age dependency would rise from $\bar{o}^1$ to o', and the youth dependency ratio would remain constant at $\bar{y}^1$. However, increases in old-age dependency drive up the tax rate in our model, given the government's balanced budget constraint. Since workers will have less disposable income, the

[19] A proof of the single crossing between the loci is available upon request.

Figure 7: Global Dynamics with Endogenous Fertility

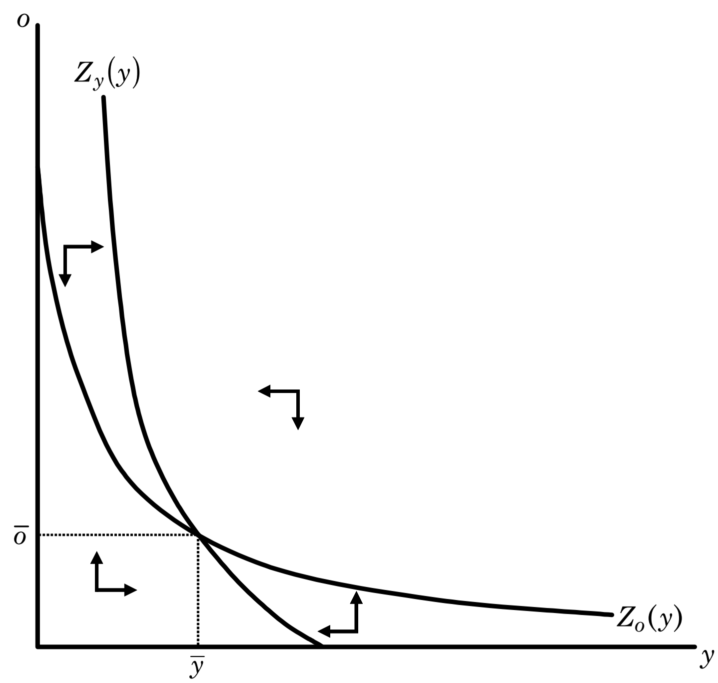
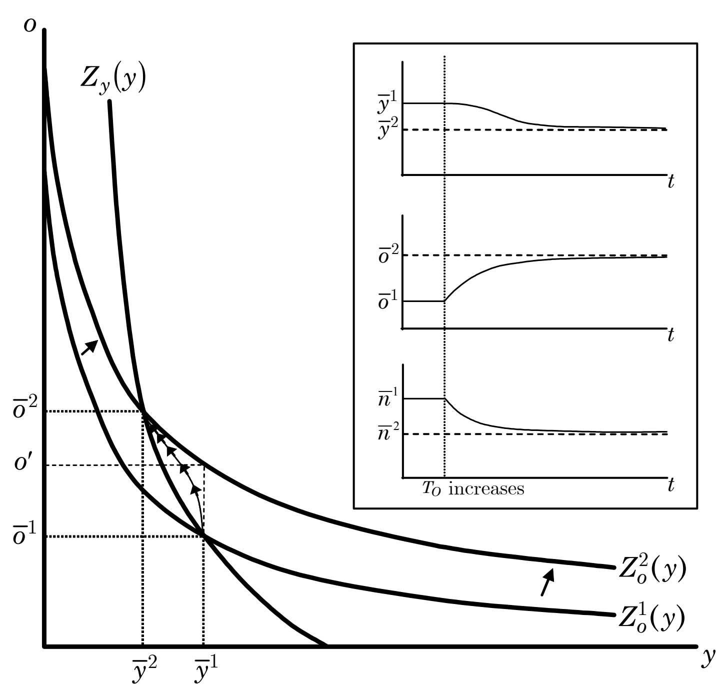


Figure 8: The Effects of a Decline in Old-age Mortality with Endogenous Fertility



rate of childbirth among the fertile will fall. We have already shown in Section 3 that declines in fertility lead, *ceteris paribus*, to increases in old-age dependency. As a result, there is a multiplier effect on declines in old-age mortality, whereby increases in old-age dependency reduce fertility, which further increase old-age dependency. However, as the fertility rate falls, so does the youth dependency ratio. The consequent reductions in public expenditures on the young acts as a brake on the feedback cycle between rising old-age dependency and falling fertility. Nonetheless, the net result is a transition to an equilibrium with an old-age dependency ratio $\bar{o}^2$ that is even higher than o'. Further the ratio of young to working falls from $\bar{y}^1$ to $\bar{y}^2$ due to the declines in fertility induced by the rise in elderly dependency.

As life expectancy among the elderly rises, the tax-minimizing fertility rate ($n_\tau^m \equiv G_\tau^m/\phi$) will increase due to the mechanism discussed in Section 3.3. However, the response of the fertile workers to an increase in life expectancy is to reduce their fertility. As a result, in the highly developed countries where $\tilde{n}$ is initially less than n_τ^m, the privately-optimal fertility response of workers to a decrease in old-age mortality ends up reducing the well-being of future workers as well as decreasing the pool of resources available to all of the other individuals in the economy.

5 Conclusion

In this paper we have analyzed the dynamic evolution of a country's population age structure and fertility rate. Our particular concern was with the feedback from population age structure to fertility via the channel of old age dependency. In a country with a high level of old-age dependency, working age individuals will see a large fraction of their labor income redistributed to the elderly. In response, working age individuals will lower fertility.

The dynamic aspects of the problem that we study are particularly important because the short and long-run effects of changes in fertility on dependency are so different. In the short run, a reduction in fertility unambiguously lowers a society's dependency burden by reducing the number of children relative to working age adults. In the long run, reductions in fertility raise a country's of old age dependency ratio, potentially undoing the reduction in youth dependency. Our calculations indicate that developed nations with the lowest levels of fertility have already passed the point where reductions in fertility raise rather than lower the long-run dependency burden.

To conduct our analysis we constructed a new continuous-time overlapping generations model that divides the population into three age groups: dependent young, working age, and dependent elderly. Individuals in each age group face constant hazards of transitioning into the next group (or into death, in the case of the elderly). The model allows us to examine the dynamic evolution of age structure of the population and the consumption dependency burden in response to changes in fertility (in the case where fertility is exogenous) or the joint evolution of fertility, age structure and dependency to exogenous shocks such as old age mortality.

Using our model, we show that for countries that are already below the level of fertility that maximizes consumption in the steady state, the actual and optimal responses of fertility to a shock to old-age survival have opposite signs. In response to greater old age survival, such countries will effectively dig themselves into a deeper demographic hole by cutting fertility in order to maintain consumption in the short run.

The model that we present is somewhat stylized in the interests of analytic tractability and ease of exposition. It could be extended along two major dimensions in order to improve its realism and forecast accuracy. First, it would be useful to expand the number of age groups beyond the three basic ones we use. Moving beyond three age groups would mean that the dynamics of the model could not be examined analytically, however, and we would be forced to take a computational approach along the lines of Auerbach and Kotlikoff (1987) and Grafenhofer et al. (2005). Second, the model could also be extended to incorporate intertemporal optimization on the part of households, which would entail shifting consumption between periods in response to anticipated changes in demographics. Such intertemporal shifting, either through investment in physical capital or purchase of foreign assets, could allow agents to save for their own old age, or the country as a whole to save for the period when there will be a high level of old-age dependency. Although this would have little effect on the steady state of the model, it would significantly influence the model's dynamics. As discussed in Cutler et al. (1990) and Elmendorf and Sheiner (2000), the ability of the economy as a whole to insulate itself against demographic shocks by building extra capital is limited by both depreciation and capital's declining marginal product. Attempts to smooth consumption in the face of demographic shocks may also result in a run-up of price of capital assets when everyone is trying to save and a corresponding asset meltdown when everyone is trying to dissave.[20]

We suspect that the extensions discussed above would not alter the fundamental predictions of our model, which are relatively grim. Demographers have been wrestling for decades with explaining below-replacement fertility in some of the world's richest countries. Low fertility is viewed as a troubling outcome in and of itself. The failure of a country's citizens to replace themselves is cited as evidence of some sort of social illness. Ironically, this hand-wringing has taken place during a period when the economic consequences of low fertility were positive. Our results suggest that over the next several decades, as the effect of low fertility on consumption possibilities turns from positive to negative, there will be a further reduction in fertility, which will, in the long run, produce

[20]See Abel (2003) and Lim and Weil (2003).

further reductions in consumption.

References

Abel, Andrew. "The Effects of a Baby Boom on Stock Prices and Capital Accumulation in the Presence of Social Security." *Econometrica* Vol. 71, No. 2 (Mar., 2003): pp. 551-578.

Auerbach, Alan J., and Laurence J. Kotlikoff. *Dynamic Fiscal Policy.* Cambridge, UK: Cambridge University Press, 1987.

Blanchard, Olivier A. "Debt, Deficits, and Infinite Horizons." *The Journal of Political Economy* Vol. 93, No. 2 (Apr., 1985): pp. 223-247.

Bloom, David E., David Canning, and Jaypee Sevilla. "Economic Growth and the Demographic Transition." Working Paper No. 8685. Cambridge, MA: National Bureau of Economic Research, 2001.

Board of Trustees, Federal Old-Age and Survivors Insurance and Disability Trust Funds. *Annual Report of the Board of Trustees of the Federal Old-Age and Survivors Insurance and Disability Insurance Trust Funds.* Washington, D.C.: Government Printing Office, 2005.

Boldrin, Michele, Mariachristina De Nardi, and Larry E. Jones. "Fertility and Social Security." Manuscript. Minneapolis, MN: University of Minnesota Department of Economics, 2005.

Bommier, Antoine, and Ronald D. Lee. "Overlapping Generations Models with Realistic Demography." *Journal of Population Economics* Vol. 16, No. 1 (Feb., 2003): pp. 135-160.

Cutler, David M., James M. Poterba, Louise M. Sheiner, Lawrence H. Summers, and George A. Akerlof. "An Aging Society: Opportunity or Challenge?" *Brookings Papers on Economic Activity* Vol. 1990, No. 1 (1990): pp. 1-73.

Diamond, Peter A. "National Debt in a Neoclassical Growth Model." *The American Economic Review* Vol. 55, No. 5 (Dec., 1965): pp. 1126-1150.

Drucker, Peter F. "The Future That Has Already Happened." *Harvard Business Review* Vol. 75, No. 5 (Sep.-Oct., 1997): pp. 20-24.

Easterlin, Richard A. *Birth and Fortune: The Impact of Numbers on Personal Welfare*, 2nd ed. Chicago, IL: University of Chicago Press, 1987.

Elmendorf, Douglas W., and Louise M. Sheiner. "Should America Save for its Old Age? Fiscal Policy, Population Aging, and National Saving." *Journal of Economic Perspectives* Vol. 14, No. 3 (Summer 2000): pp. 57-74.

Gertler, Mark. "Government debt and social security in a life-cycle economy.' *Carnegie-Rochester Conference Series on Public Policy* Vol. 50 (Jun., 1999): pp. 61-110.

Grafenhofer, Dominik, Christian Jaag, Christian Keuschnigg, and Mirela Keuschnigg. "Probabilistic Aging." Manuscript. St. Gallen, Switzerland: University of St. Gallen IFF, 2005.

Kohler, Hans-Peter, Francesco C. Billari, and Jose Antonio Ortega. "The Emergence of Lowest-Low Fertility in Europe during the 1990s." *Population and Development Review* Vol. 28, No. 4 (Dec., 2002): pp. 641-680.

Lee, Ronald, and Lawrence R. Carter. "Modeling and Forecasting U.S. Mortality." *Journal of the American Statistical Association* Vol. 87, No. 419 (Sep., 1992): pp. 659-671.

Lee, Ronald D., and Ryan D. Edwards, "The Fiscal Impact of Population Change" in Jane Sneddon Little and Robert K. Triest, eds., *Seismic Shifts: The Economic Impact of Demographic Change*. Boston, MA: Federal Reserve Bank of Boston Conference Series, No. 46, June, 2001, pp. 189-219.

Lim, Kyung-Mook, and David N. Weil, "The Baby Boom and the Stock Market Boom." *Scandinavian Journal of Economics*, Vol. 105, No. 3 (Sep., 2003): pp. 359-377.

Mason, Andrew, Ronald Lee, An-Chi Tung, Mun-Sim Li, and Tim Miller. "Population Aging and Intergenerational Transfers: Introducing Age into National Accounts." Paper presented at the Population Association of America Annual Meeting, 2005.

Micevska, Maja B., and Paul J. Zak. "What Accounts for the Emergence of Malthusian Fertility in Transition Economies?" Working Paper No. 2002-01. Claremont, CA: Claremont McKenna College, Department of Economics, 2002.

Munnell, Alicia H., Robert E. Hatch, and James G. Lee. "Why is Life Expectancy So Low in the United States?" *Issues in Brief* No. 21. Boston: MA: Boston College, Center for Retirement Research, 2004.

Organization for Economic Cooperation and Development [OECD]. *Society at A Glance: OECD Social Indicators – 2005 Edition.* Paris, France: OECD, 2004.

Samuelson, Paul A. "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money." *The Journal of Political Economy* Vol. 66, No. 6 (Dec., 1958), pp. 467-482.

United Nations. Department of Economic and Social Affairs, Population Division. *World Population Prospects: The 2004 Revision.* New York, United Nations, forthcoming.

___________. Department of Economic and Social Affairs, Statistics Division. *Demographic Yearbook 1997 – Historical Supplement.* (CD-ROM special issue.) New York: United Nations, 2000.

___________. ___________. *Demographic Yearbook 2002.* New York: United Nations, 2005.

Weil, David N. "Population Growth, Dependency, and Consumption." *The American Economic Review* Vol. 89, No. 2, Papers and Proceedings of the One Hundred Eleventh Annual Meeting of the American Economic Association (May, 1999): pp. 251-255.